AD-A213 788

DTIC FILE COPY

(2)

AD

TECHNICAL REPORT ARCCB-TR-89020

THE INFLUENCE OF TRANSIENT FLEXURAL WAVES ON DYNAMIC STRAINS IN GUN TUBES

THOMAS E. SIMKINS

OCT 25 1989

AUGUST 1989

US ARMY ARMAMENT RESEARCH,
DEVELOPMENT AND ENGINEERING CENTER
CLOSE COMBAT ARMAMENTS CENTER
BENÉT LABORATORIES
WATERVLIET, N.Y. 12189-4050

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

89 10 25 025

DISCLAIMER

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

The use of trade name(s) and/or manufacturer(s) does not constitute an official indorsement or approval.

DESTRUCTION NOTICE

For classified documents, follow the procedures in DoD 5200.22-M, Industrial Security Manual, Section II-19 or DoD 5200.1-R, Information Security Program Regulation, Chapter IX.

For unclassified, limited documents, destroy by any method that will prevent disclosure of contents or reconstruction of the document.

For unclassified, unlimited documents, destroy when the report is no longer needed. Do not return it to the originator.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ARCCB-TR-89020	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) THE INFLUENCE OF TRANSIENT FLEXURAL WAVES ON DYNAMIC STRAINS IN GUN TUBES		5. TYPE OF REPORT & PERIOD COVERED Final 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Thomas E. Simkins		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS U.S. Army ARDEC Benet Laboratories, SMCAR-CCB-TL Watervliet, NY 12189-4050		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS AMCMS No. 6111.02.H610.0 PRON No. 1A94Z9CANMSC
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army ARDEC Close Combat Armaments Center Picatinny Arsenal, NJ 07806-5000		12. REPORT DATE August 1989 13. NUMBER OF PAGES 34
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Waves
Wave Propagation
Transient Vibrations
Steady-State Vibrations
Flexural Vibrations, ~~of a~~ Cylinder

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Transient waves or vibrations can be excited in several ways as ballistic pressure traverses the length of a gun tube. This report shows that despite their diversity, these transients will constructively interfere with the steady-state deformation in a highly predictable way to cause alternating regions of high circumferential strain along the tube. The method of stationary phase plays a key role in the analysis.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

TABLE OF CONTENTS

TABLES

LIST OF ILLUSTRATIONS

A-1

INTRODUCTION AND SYNOPSIS

The study of flexural wave propagation in gun tubes continues to be interesting and pertinent to the goals of this laboratory. As reported in Reference 1, the steady-state theory of flexural waves predicts circumferential strains in excellent agreement with measured values as long as the projectile velocities are very close to critical. At subcritical velocities, however, the agreement is not quite as good and it was this observation that motivated the work reported herein. As a brief review, the general appearance of this steady deformation for two different velocities of the moving ballistic pressure is shown in Figures 1a and 1b. (The ordinates in all figures except Figure 3 are normalized with respect to the static value as calculated under the Lamé conditions, i.e., the deformation of the wall of an infinitely long and uniform cylinder exposed to a constant internal pressure.) The dramatic increase in maximum strain as the velocity approaches critical is evident in Figure 1b. A comparison of measured strains with the predicted values from the steady-state theory (Figure 2) nevertheless shows the predicted values to be too low except when the velocity is nearly critical. Since the measured strains are greater than the predicted values at projectile velocities considerably less than critical, such strains are not only of concern in the latest generation gun tubes, but also in conventional tubes in which critical projectile velocities are generally not attained. This report addresses one possible cause of these strains.

ABAQUS (ref 2) results by A. Gabriele (ref 3) gave the first hint that the steady-state theory of flexural waves, successful in predict ıg the dramatic

[1]T. E. Simkins, "Resonance of Flexural Waves in Gun Tubes," ARCCB-TR-87008, Benet Laboratories, Watervliet, NY, July 1987.

[2]"ABAQUS Finite Element Code," Version 4.5(a), Hibbitt, Karlsson, and Sorenson, Inc., 1985.

[3]A. Gabriele, Private Communication, U.S. Army ARDEC, Benet Laboratories, Watervliet, NY, October 1988.

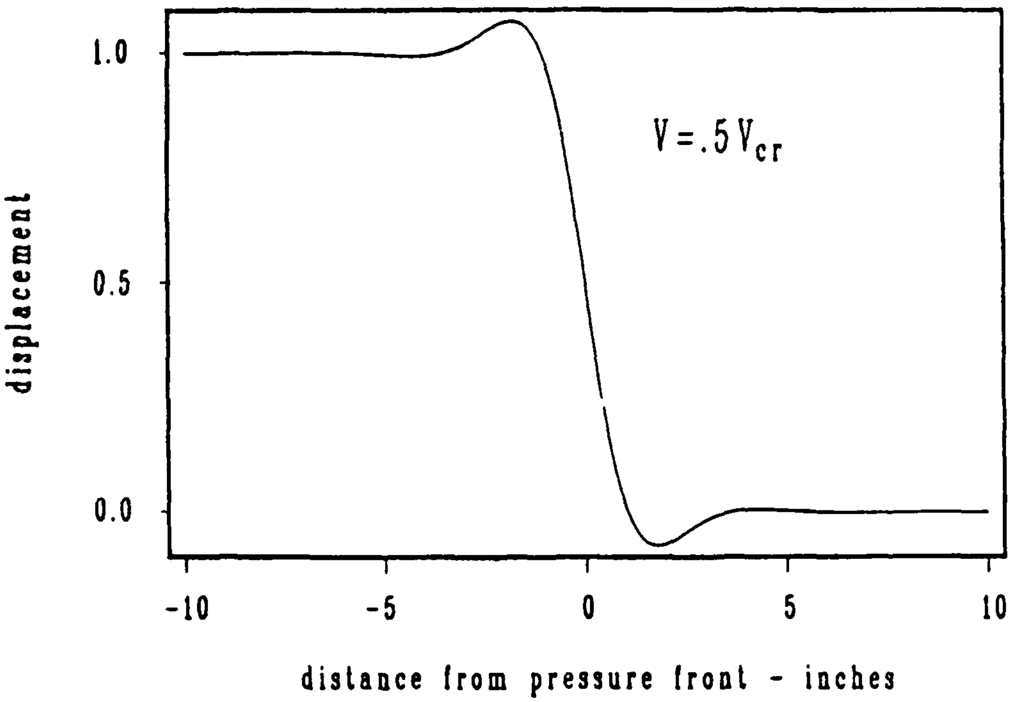

Figure 1a

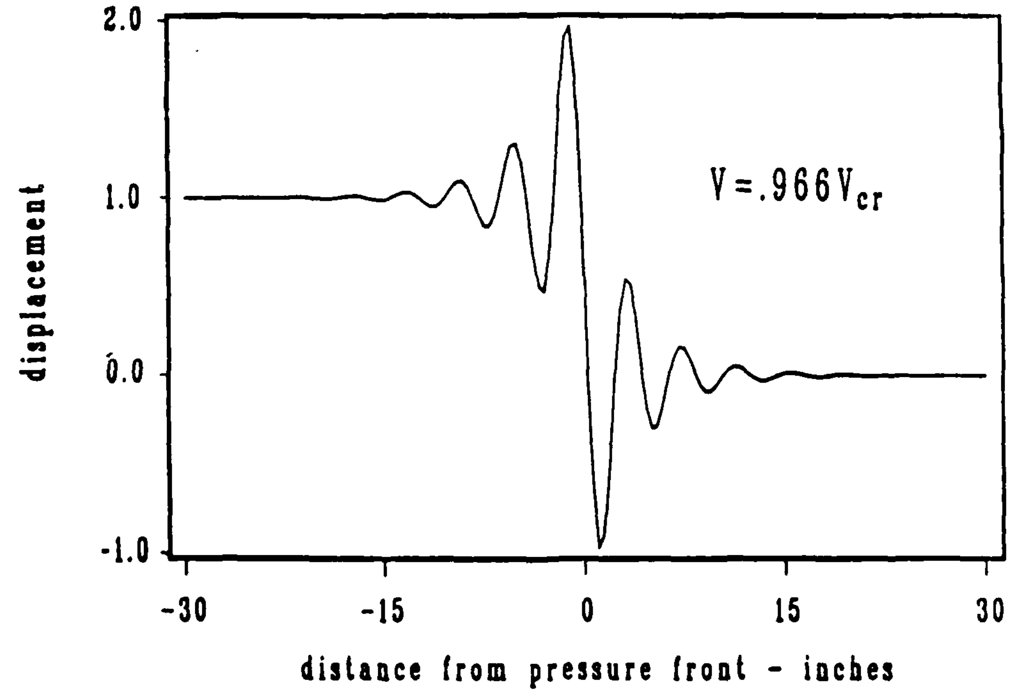

Figure 1b

Figure 1. Radial displacement of the wall of a uniform tube of infinite length at two different pressure velocities, 120-mm XM25.

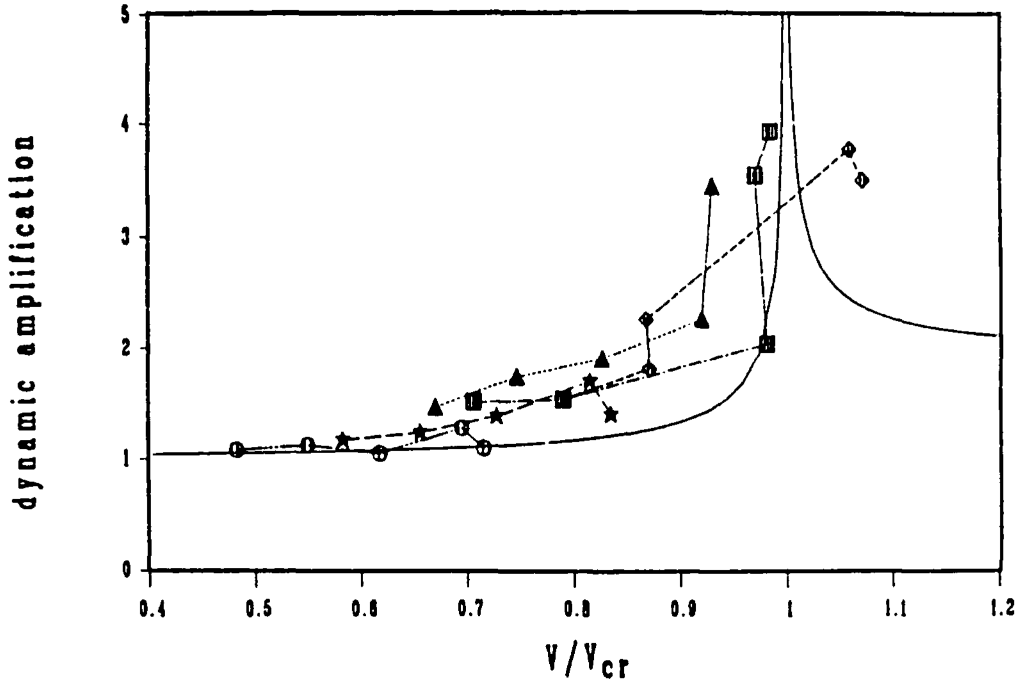

Figure 2. Comparison of maximum measured strains with those predicted by the steady-state theory, 120-mm XM25.

increase of tube strains at projectile velocities near critical, may nevertheless predict strain values which are too low. The ABAQUS strains at some locations along the tube at projectile passage were significantly higher as were the measured values. Non-uniformities in the tube wall thickness in the axial direction were included in these finite element models as well as the variable ballistic pressure and the velocity with which this pressure travels along the tube. This is in contrast to the continuum model in Reference 1 which assumes a constant pressure moving at constant velocity in an infinite tube of uniform wall thickness. The degree to which variability in each of these quantities plays a part in creating the higher strains from the ABAQUS model and those measured along the real tube will be a goal for some time to come. However, the work reported herein moved forward on the premise that a major effect of these variabilities is the production of transient vibrations or waves which, together with the steady solution established in Reference 1, will help explain the

[1]T. E. Simkins, "Resonance of Flexural Waves in Gun Tubes," ARCCB-TR-87008, Benet Laboratories, Watervliet, NY, July 1987.

higher strain values. For example, intuition tells us that as the moving ballistic pressure suddenly encounters an abrupt change in the tube's cross-sectional area, a transient vibration will be produced--an effect clearly beyond the scope of Reference 1, but certain to occur in the ABAQUS models. Just how abrupt this change in area must be to excite a significant transient is a matter for future study. One might also suspect that the sudden rise of the ballistic pressure following ignition and/or its acceleration along the tube might generate transients.

Although the ABAQUS models without question mirror the actual gun tube in most respects, there is one feature present in the ABAQUS models that in general does not mimic the actual gun tube. Because of the impracticality of modeling the entire tube, these ABAQUS models typically consider only a segment of the tube--usually one which ends at the muzzle. Since this segment of the tube is, by necessity, considered a free body--completely disconnected from the remainder of the tube--an unrealistic entrance condition is created. That is, in the ABAQUS models, the moving pressure suddenly appears at the entrance of an undisturbed tube, a condition not present in the real tube. It is apparent that this excites an additional transient vibration in the model not present in the actual gun tube. In this report, the contribution of this entrance transient to the overall predicted deformation of the tube is shown to be significant.

As things turned out, the presence of the artificial entrance condition in the ABAQUS models was fortuitous because of the ease with which this transient could be studied. The final outcome was a firm understanding of how transients <u>in general</u> act to increase tube strains. As previously mentioned, there are several processes during the firing of an actual gun tube which are likely to

[1]T. E. Simkins, "Resonance of Flexural Waves in Gun Tubes," ARCCB-TR-87008, Benet Laboratories, Watervliet, NY, July 1987.

produce transients, some of which are represented in the ABAQUS models. The general understanding achieved through the study of the entrance transient tells us qualitatively what to expect in the way of strain increases within the tube.

ABAQUS RESULTS

Non-Uniform Wall, Variable Pressure and Velocity

The main item of engineering interest in the ABAQUS results is the maximum strain experienced at each axial location during the time that the pressure moves from entrance to exit. A portion of a typical plot of these maximum strains as obtained by Gabriele i. shown in Figure 3.

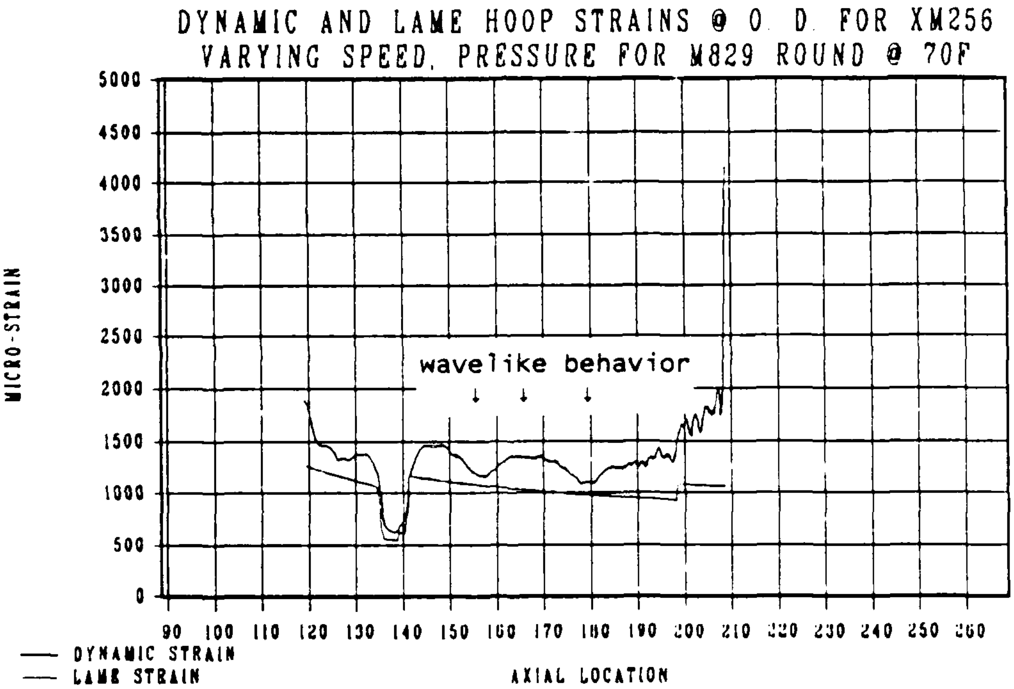

Figure 3. Maximum circumferential strain prior to shot ejection - XM256. ABAQUS (Gabriele).

Uniform Wall, Constant Pressure and Velocity

A prominent characteristic in Figure 3 is the wavelike appearance of the maximum strain distribution. To more easily investigate the cause of this waviness, a new ABAQUS model was requested. In this model, the pressure and velocity were constant and the tube wall had uniform thickness. Figures 4a and 4b show typical distributions of maximum strains for this model resulting from pressures moving at two different velocities as computed by ABAQUS.

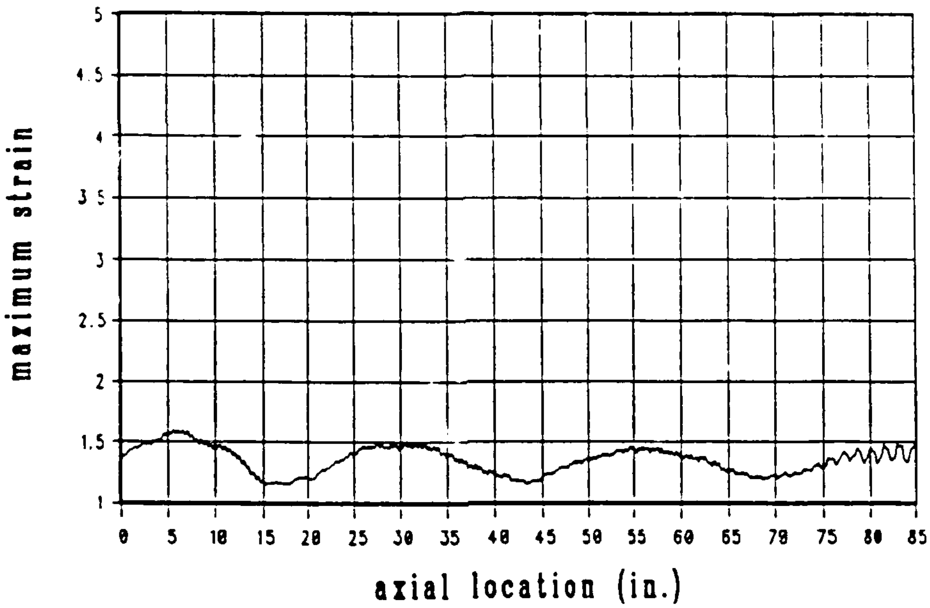

Figure 4a. Velocity of moving pressure = 4550 ft/sec.

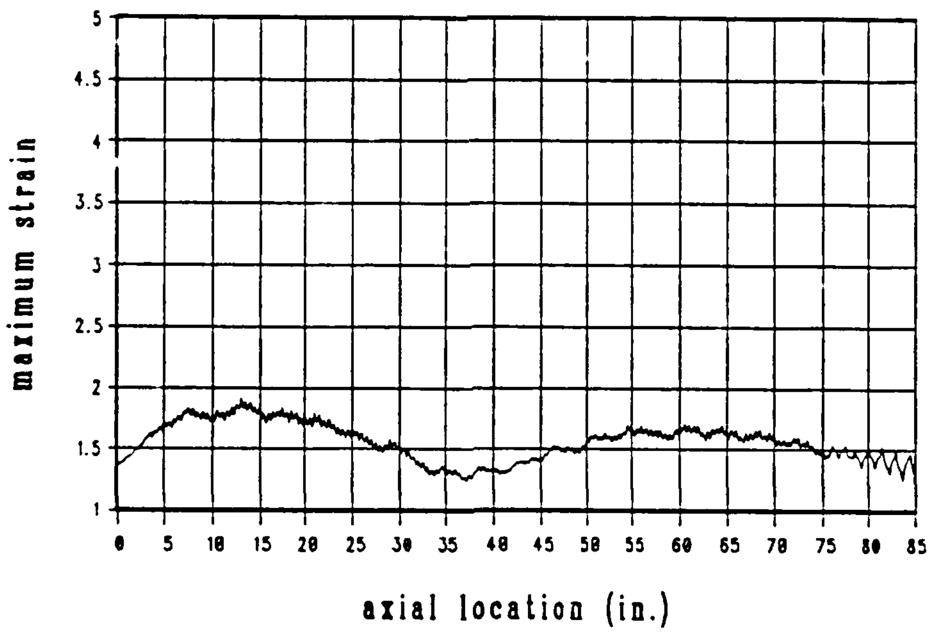

Figure 4b. Velocity of moving pressure = 4818 ft/sec.

Figure 4. Maximum circumferential strain prior to shot ejection for two different pressure velocities. ABAQUS (Gabriele).

The wavelike character in these figures is very pronounced and constitutes the main difference between these ABAQUS strains and those calculated from the steady-state deformation caused by the same moving pressure in an infinite tube. It is also observed that the wave attentuates with distance along the tube and has a longer wavelength at the nigher velocity. In comparing the strain levels of Figure 4 with those predicted by the steady-state theory, it is found that the latter are in excellent agreement with the <u>average</u> of the former, i.e., the essential difference between the two appears to be a decaying periodic function.

The cause of the wavelike variation in the maximum ABAQUS strains evidently has little to do with variable velocity, pressure, or wall thickness since this variation appears even when these are made constant. Other than the finiteness of the tube length itself, the only other possible causes for this variation are the entrance transient mentioned previously or perhaps other non-steady motions which arise due to the motion of the pressure. It is shown in this report, however, that no unsteady motions other than the entrance transient are present.

A SIMPLIFIED MODEL FOR SYMBOLIC SOLUTION

At this point in the investigation, ABAQUS or other strictly numerical methods of solution provided no further insight. What was needed was a solution in symbolic or analytical form from which qualitative information might be obtained. For this purpose, a further simplification was made to the model. By assuming that the wall of the tube is very thin, the problem can be formulated as a simple boundary value problem in dynamics which can be solved either by Fourier transforms or by Fourier series. The advantage of either of these methods is that the transient term(s) and the steady-state terms appear as separate analytical expressions. The interaction of these terms can then be explored.

The simplest equation governing the axisymmetric motion of a thin-walled cylindrical shell subjected to a step function of pressure moving at constant velocity can be written as follows (ref 1):

$$D \frac{\partial^4 w}{\partial x^4} + \frac{Eh}{R^2} w + \rho h \frac{\partial^2 w}{\partial t^2} = Q(1-H(x-V_p t)) \qquad (1a)$$

where Q is a constant and represents the magnitude of the moving pressure and H is the Heaviside step function:

$$\begin{aligned} H(x-V_p t) &= 0 \quad x < V_p t \\ &= 1 \quad x > V_p t \end{aligned}$$

In this equation, w is the radial displacement of the median surface of the cylindrical shell located at a distance R from the central axis; h is the shell thickness and is assumed to be small compared to R; ρ is the mass density of the shell material; $D = Eh^3/12(1-\nu^2)$; E is Young's modulus of elasticity; ν is Poisson's ratio; and V_p is the velocity of the moving pressure, assumed to be constant. With a different interpretation of the coefficients, Eq. (1a) is equivalent to the equation governing the motion of a Bernoulli-Euler beam on an elastic foundation, and accordingly, shear deformation and rotatory inertia are ignored.

As in the ABAQUS models, the tube is at rest initially and the radial displacement is constrained to be zero at each end, i.e.,

$$w(x,0) = \dot{w}(x,0) = 0 \qquad (1b)$$

$$w(0,t) = w(L,t) = w''(0,t) = w''(L,t) = 0 \qquad (1c)$$

[1]T. E. Simkins, "Resonance of Flexural Waves in Gun Tubes," ARCCB-TR-87008, Benet Laboratories, Watervliet, NY, July 1987.

As previously mentioned, either Fourier series or Fourier transforms can be used with advantage. Fourier series happens to be the first way the author approached the problem and has the advantage that the individual terms of the resulting solution are more directly analogous to those of the familiar single-degree of freedom system. However, a complete interpretation of these results requires at least a partial use of Fourier transforms. On the other hand, exclusive use of the Fourier transform method, with the help of an asymptotic approximation, provides a closed-form symbolic solution. Both solutions are given in this report.

Prior to solving the system (Eq. (1)), it is worth reviewing the meanings of the terms 'steady-state' and 'transient' as they arise in connection with a simple problem involving a single-degree of freedom. Therefore, consider the following problem:

Equation of motion:

$$\ddot{u} + \omega^2 u = f_0 \cos(\omega_f t) + f_1 t \tag{2a}$$

Initial conditions:

$$u(0) = u_0 \quad , \quad \dot{u}(0) = \dot{u}_0 \tag{2b}$$

Solution:

$$u = u_0 \cos(\omega t) + \frac{\dot{u}_0}{\omega} \sin(\omega t) - \frac{f_1}{\omega^2} \sin(\omega t)$$

$$\frac{-f_0}{\omega^2 - \omega_f^2} \cos(\omega t) + \frac{f_0}{\omega^2 - \omega_f^2} \cos(\omega_f t) + \frac{f_1}{\omega^2} t \tag{2c}$$

The first two terms of this solution represent the motion in existence at $t = 0$ and have the natural frequency of the system. The third and fourth terms also have the natural frequency of the system, but are associated with the applied forces. These four terms are the well-known 'transients' of the solution, so named because they die out if damping is present. The remaining terms comprise the steady-state solution, so named because they do not die out if damping is present. The reason for including the force $f_1 t$ as well as the more traditional harmonic force is simply to show that unless $f_1 = 0$, there is nothing 'steady' about the steady-state solution. Thus, the word 'steady' cannot always be taken literally. Having made this observation, we henceforth let $f_1 = 0$, whereupon the solution becomes simply

$$u = u_0 \cos(\omega t) + \frac{\dot{u}_0}{\omega} \sin(\omega t) - \frac{f_0}{\omega^2 - \omega_f^2} \cos(\omega t) + \frac{f_0}{\omega^2 - \omega_f^2} \cos(\omega_f t) \quad (2d)$$

Now physically, we know that for arbitrary initial conditions the system cannot assume a steady-state response instantaneously. There must be a transition motion which takes it from its initial state $(u_0, \dot{u}_0)$ to the steady state. The third transient in the solution (Eq. (2d)) accomplishes this. One might therefore refer to this term as a 'transition transient.' Similarly, the deformation resulting from a moving pressure suddenly entering a tube cannot start out as steady state, but must also have a 'transition transient' present in its solution as well as transients representative of the initial conditions.

Solution (2d) above also shows that the transition transient, in the absence of damping, periodically interferes with the deformation represented by the steady-state solution (beats) and, like the steady solution, becomes unbounded as the forcing frequency approaches the natural frequency of the system. Similar, but not identical, results are shown for the problem of the tube subjected to a moving pressure.

Finally, it is worth remembering that each transient constitutes a possible <u>free motion</u> of the system. This viewpoint will prove useful in studying the problem of interest since the analysis of freely propagating disturbances is well developed.

SOLUTION BY FOURIER SERIES

One way to arrive at a Fourier series solution to Eq. (1) is to use the sinusoidal vibrational mode shapes as basis functions in the Galerkin procedure. Thus, a solution of the following form is assumed:

$$w(x,t) = \sum_{j=1}^{\infty} \phi_j(t) \sin(j\pi x/L) \tag{3}$$

By inspection, it is clear that this solution will satisfy the boundary conditions (Eq. (1c)) and that the initial conditions (Eq. (1b)) require that

$$\phi_j(0) = \dot{\phi}_j(0) = 0$$

It remains only to determine the time variant coefficients $\phi_j(t)$. To do this, Eq. (1a) is multiplied by the arbitrary variation $\delta w = \delta\phi_j \sin(j\pi x/L)$ and integrated over the interval (0,L). Using the boundary conditions and the usual orthogonality condition

$$\int_0^L \sin\left(\frac{i\pi x}{L}\right)\sin\left(j\,\frac{\pi x}{L}\right)dx = \begin{cases} 0 & i \neq j \\ L/2 & i = j \end{cases}$$

results finally in the expression

$$\tfrac{1}{2} \sum_{j=1}^{\infty} \delta\phi_j\left\{D\left(\frac{j\pi}{L}\right)^4\phi_j + \frac{Eh}{R^2}\,\phi_j + \rho h\ddot{\phi}_j - \frac{2}{L}\int_0^L Q(1-H(x-V_pt))\sin\left(j\,\frac{\pi x}{L}\right)dx\right\} = 0$$

Since each $\delta\phi_j$ is arbitrary and independent, each term of this series must vanish with the result

$$\ddot{\phi}_j + p_j^2\phi_j = \frac{2Q}{j\rho h\pi}\left(1-\cos\left(\frac{j\pi V_p t}{L}\right)\right) \equiv F_j(t) \quad , \quad j = 1,2,\ldots\infty \tag{4}$$

where

$$p_j^2 = \frac{a^2\pi^4}{L^4}(j^4+\beta) \;;\quad a^2 = D/\rho h \;;\quad \beta = \frac{Eh}{R^2 D}\left(\frac{L}{\pi}\right)^4$$

The Duhamel form of solution to Eq. (4) is

$$\phi_j(t) = \int_0^t \frac{F_j(\tau)}{p_j} \sin(p_j\ (t-\tau))d\tau$$

which results in

$$\phi_j(t) = \frac{f_j}{p_j^2(p_j^2-b^2j^2)} \{b^2j^2(\cos(p_jt)) - p_j^2(\cos(jbt)) + p_j^2 - b^2j^2\} \qquad (5)$$

where

$$f_j = 2Q/j\rho h\pi \quad ; \quad b = \pi V_p/L$$

The ϕ_j's from Eq. (5) are then substituted into the assumed solution (Eq. (3)) to arrive at the final series solution for the displacement of the tube wall in space and time. Figure 5 is a plot of the tube shape at the instant when the load reaches mid-tube. The coefficients of the shell equation (Eq. (1a)) for this illustration correspond to a 120-mm tube of uniform wall

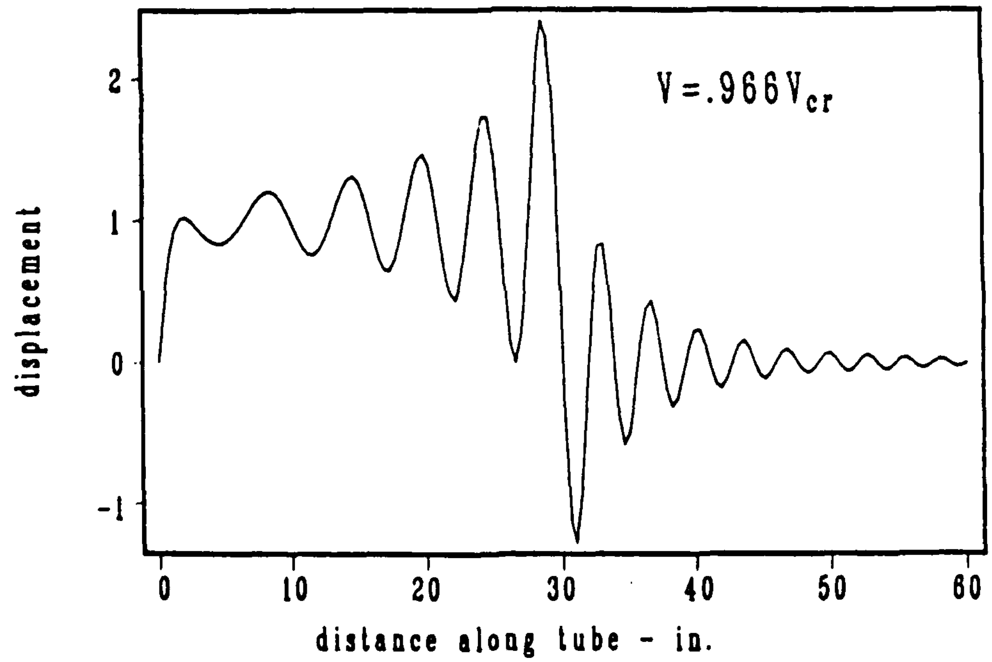

Figure 5. Radial displacement of tube wall when pressure has travelled 50 percent of the tube length--by Fourier series.

thickness and a length of 60 inches. Tube properties are typical of the 120-mm XM25, as reported in Reference 1, i.e.,

$$h = 0.508 \text{ in.} \qquad E = 30.3\text{e}6 \text{ lb/in.}^2$$

$$R = 2.616 \text{ in.} \qquad \nu = 0.3$$

$$\rho = 0.0007365 \text{ slugs/in.}^3 \qquad Q = Eh/R^2$$

Choosing $Q = Eh/R^2$ effectively normalizes the displacement with respect to the static deformation under Lamé conditions. The velocity of the moving pressure was arbitrarily chosen to be 96.6 percent of the critical value.

The superiority of the Fourier series solution, Eq. (5), to one achieved using finite basis functions (finite elements, finite differences, etc.) is that it allows the transient terms to be easily distinguished from the steady-state solution just as in (Eq. (2d)). Note that since the initial conditions are zero, the only the remaining transient in the solution expression for the ϕ_j is the transition transient (the term in $\cos(p_j t)$). This is the transition transient part of the solution for the $\phi_j(t)$. The transition transient part of the solution for the displacement $w(x,t)$ is then formed by multiplying this term by $\sin(j\pi x/L)$ and summing. Now the question can be asked: If one ignores the transient in Eq. (5), will the remaining terms (when multiplied by $\sin(j\pi x/L)$ and summed) yield a solution which looks anything like the steady solution for an infinite tube? The answer is shown in Figure 6a. A comparison with the corresponding steady deformation of a tube of infinite length (Figure 6b) shows that except near the ends of the tube, the solutions will be virtually the same. (As the velocity of the moving pressure approaches the critical value, however, the influence of the boundary extends farther into the tube.)

[1]T. E. Simkins, "Resonance of Flexural Waves in Gun Tubes," ARCCB-TR-87008, Benet Laboratories, Watervliet, NY, July 1987.

It can therefore be concluded that in regions of the tube not too close to the ends, the complete solution to the problem consists of the sum of the steady solution corresponding to a moving pressure in a tube of infinite length and the transition transient resulting from the sudden entrance of pressure. No other motions are generated by the moving pressure. (This is probably a consequence of the constant velocity of the moving pressure.)

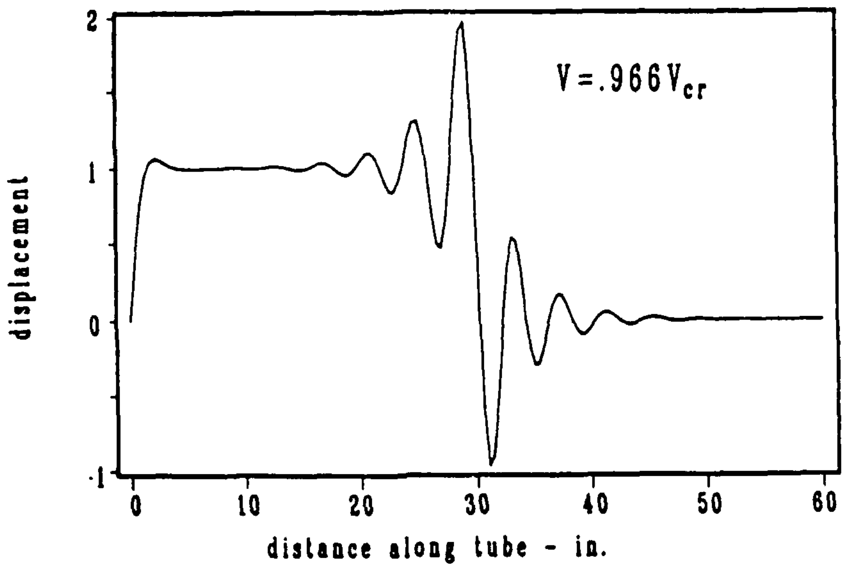

Figure 6a. Fourier solution for the deformation of a tube of finite length neglecting transient term.

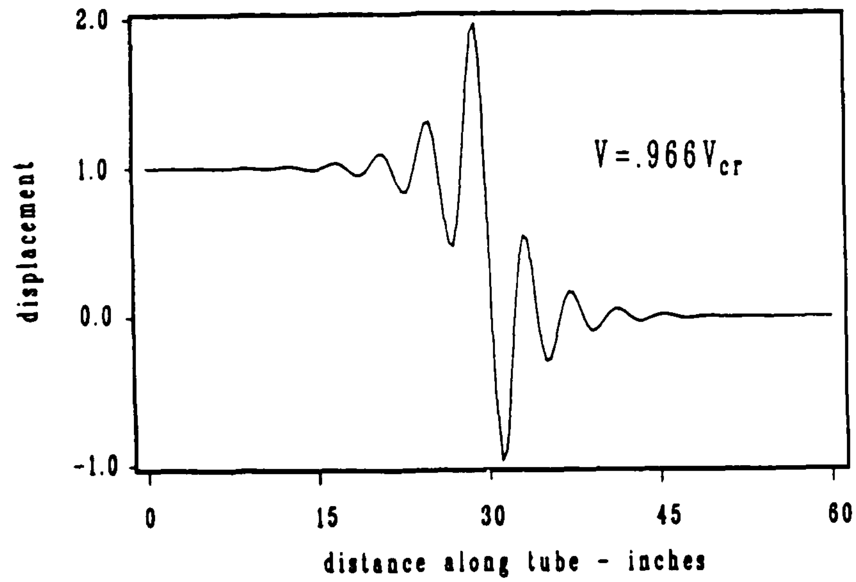

Figure 6b. Steady-state deformation of a tube of infinite length.

Comparing Figures 5 and 6, the effect of the transient at the instant the load reaches mid-length is to increase the maximum wall displacement (and therefore the circumferential stress and strain) by about 17 percent beyond that predicted by the steady-state solution in Figure 6b. This is not necessarily the time at which the largest displacement will appear, however, and it is apparent that it is the interaction between the steady solution and the transition transient which needs to be understood. For this purpose the transition transient is viewed as a free, unforced motion of the tube wall because the analysis of freely propagating disturbances is well developed. This motion will result, for example, from certain initial conditions imposed on a tube in the absence of any pressure. Thus, one can consider the following problem of free vibration:

$$\text{d.e.} \quad D \frac{\partial^4 w}{\partial x^4} + \frac{Eh}{R^2} w + \rho h \frac{\partial^2 w}{\partial t^2} = 0 \tag{6a}$$

$$\text{b.c.} \quad w(0,t) = w(L,t) = w''(0,t) = w''(L,t) = 0 \tag{6b}$$

$$\text{i.c.} \quad w(x,0) = \sum_{j=1}^{\infty} \phi_j(0) \sin(j\pi x/L), \; \dot{w}(x,0) = 0 \tag{6c}$$

where

$$\phi_j(0) = b^2 j^2 f_j / p_j^2 (p_j^2 - b^2 j^2) \quad ; \quad \dot{\phi}_j(0) = 0$$

Solution:

$$w(x,t) = \sum_{i=j}^{\infty} \phi_j(0) \cos(p_j t) \sin(j\pi x/L) \tag{7}$$

which is, by intent, the transition transient of the corresponding forced motion solution (Eq. (5)). This transient is shown in Figures 7a and 7b for two different times. The effect of dispersion is evident because the deformation modulates in wavelength along the tube.

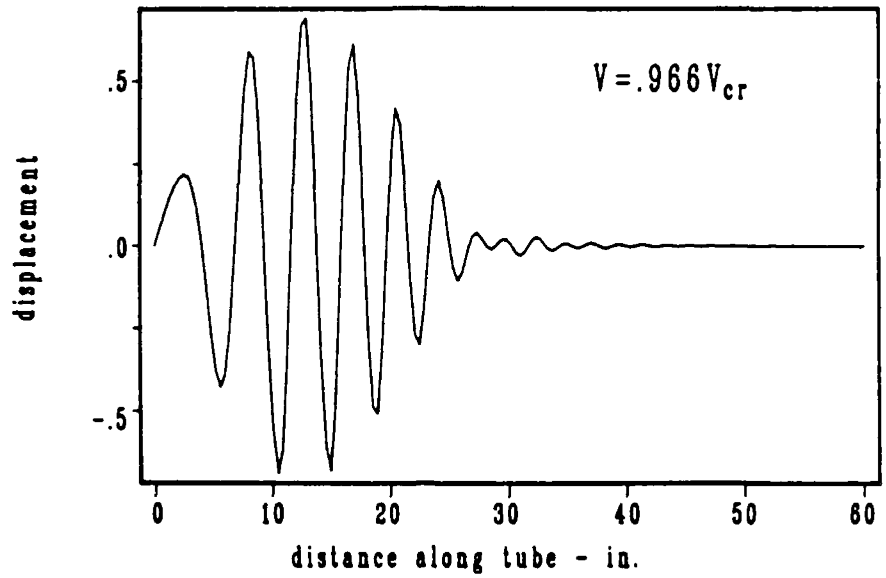

Figure 7a. The transition transient when the pressure has travelled 25 percent of the tube length--by Fourier Series.

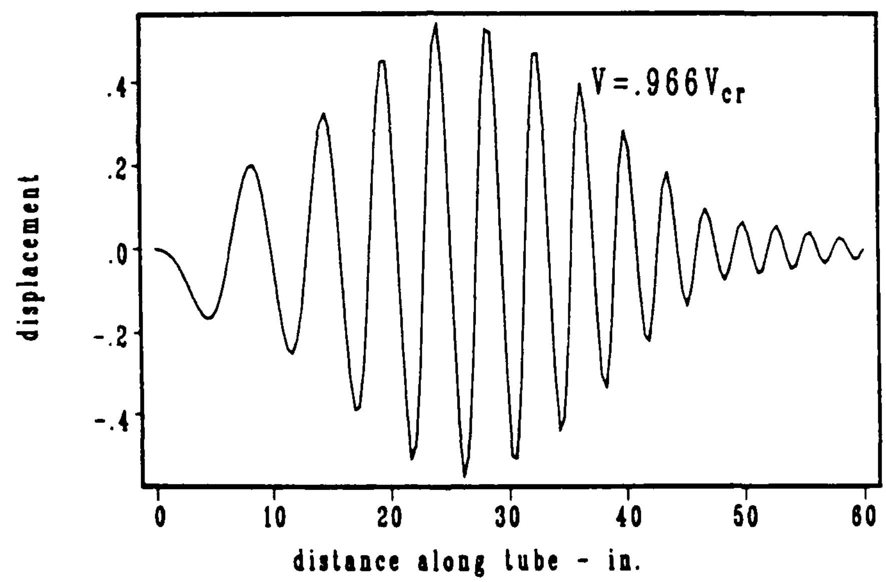

Figure 7b. The transition transient when the pressure has travelled 50 percent of the tube length--by Fourier Series.

Now the primary concern is to learn how this transition signal interacts with the steady solution to periodically increase and decrease the deformation as in Figure 4. It would be very helpful, therefore, if a closed-form solution (symbolic) could be achieved instead of Eq. (7). For this purpose, some results achieved by the use of Fourier transforms are most suitable. The transformed solution to the problem defined by Eq. (6) is easy to obtain, but to arrive at the actual displacement of interest, this expression must be inverted. Unfortunately, the required inverse can only be obtained numerically which is of no help in understanding the result. Fortunately, however, an asymptotic evaluation of the inverse is possible and the result is very informative. The method used to obtain this asymptotic evaluation is the method of stationary phase (ref 4) which shows that if a function h(k) has a stationary value at the point $k = k_0$, then the following integral can be evaluated asymptotically (as t becomes infinite):

$$y(t) = \int_a^b \Phi(k) e^{ith(k)} dk \tag{8}$$

Asymptotic evaluation by stationary phase gives

$$y(t) = \Phi(k_0) \sqrt{\frac{2\pi}{t|h''(k_0)|}} \exp\left(-ih(k_0)t - \frac{i\pi}{4} \operatorname{sgn} h''(k_0)\right) \tag{9}$$

That is, the main contribution to the integral (Eq. (8)) is from the neighborhood of $k = k_0$. Otherwise the contributions oscillate rapidly and make little net contribution.

For the problem specified by Eq. (7) above, the solution by Fourier transforms can be written (ref 5):

$$w(x,t) = \sqrt{\frac{1}{2\pi}} \operatorname{Im}\left\{\int_0^\infty \Phi(k)\exp[i(kx+W(k)t)]dk + \right.$$

$$\left. \int_0^\infty \Phi(k)\exp[i(kx-W(k)t)]dk\right\} \tag{10}$$

[4] Karl F. Graff, Wave Motion in Elastic Solids, Ohio State University Press, 1975, pp. 65-66.

[5] G. B. Whitham, Linear and Nonlinear Waves, John Wiley and Sons, New York, 1974, p. 373.

where $\omega = W(k)$ is the so-called dispersion relation expressing the circular frequency ω as a function of the wave number, k. W(k) characterizes the system properties, while the initial conditions (Eq. (6c)) are represented in $\Phi(k)$. For the system represented by Eqs. (6a) through (6c)

$$\Phi(k) = \sqrt{\frac{2}{\pi}} \int_0^\infty w(x,0)\sin(kx)dx \tag{11}$$

and from Reference 1

$$W(k) = \sqrt{\frac{Eh/R^2 + Dk^4}{\rho h}} \tag{12}$$

Putnick (ref 6) has shown that the first term of Eq. (10) contributes nothing since there is no value of $k > 0$ for which the exponent is stationary. Applying Eq. (8) to the second term

$$h(k) = kx/t - W(k) \tag{13}$$

In order for h(k) to be stationary at k_0

$$h'(k_0) = x/t - W'(k_0) = 0 \tag{14}$$

or

$$W'(k_0) = x/t$$

However, $W'(k_0)$ is simply the group velocity (ref 7) associated with the wave number k_0. Finally, the solution to the system represented by Eq. (7), provided t is large enough, is

$$w(x,t) = -\Phi(k_0)\sqrt{\frac{2\pi}{tW''(k_0)}}\sin(W(k_0)t - k_0x + \pi/4) \tag{15}$$

[1]T. E. Simkins, "Resonance of Flexural Waves in Gun Tubes," ARCCB-TR-87008, Benet Laboratories, Watervliet, NY, July 1987.

[6]L. Putnick, Private Communication, U.S. Army ARDEC, Benet Laboratories, Watervliet, NY, October 1988.

[7]G. B. Whitham, Linear and Nonlinear Waves, John Wiley and Sons, New York, 1974, p. 10.

Equation (15) is the asymptotic evaluation of the transition transient in closed form and is loaded with information. Basically this stationary-phase result states that the dominant part of the disturbance that arrives at a particular point x at time t will have travelled with a group velocity x/t and will consist of the dominant wave number k_0 determined from Eq. (14).

For example, if an observer travels at some constant velocity (say V_0), then at the location $x = V_0 t$ the observer will perceive the transition transient as a single wave (and hence a single frequency) at all times. This frequency and wave number correspond to a wave whose group velocity is V_0. Now suppose $V_0 = V_p$, the velocity of the moving pressure and hence the translational velocity of the steady deformation. As Figure 1 shows, the maximum of this steady deformation is located a short distance behind the pressure front. If this distance is d, the transition transient at this location will effectively consist of a single wave with group velocity $V_p - d/t$. Since t has been assumed large, d is small compared with $V_p t$ and the group velocity will be very nearly V_p. The quantity d/t will henceforth be neglected.

It has been shown that at the location of the maximum steady-state deformation, the transient consists of a signal having one and only one frequency and wave number. The total interaction between the steady signal and the transient signal is therefore simple harmonic with an amplitude given by Eq. (15) which decays as $t^{-1/2}$. At the location of the maximum steady-state solution, $x = V_p t$, and thus there will be a spatially decaying periodic variation in the maximum total deformation along the tube. The wavelength of this variation is easily obtained from Eq. (15) by substituting $t = x/V_p$ and examining the argument of the oscillatory term, i.e.,

$$\sin(W(k_0)t - k_0 x + \pi/4)\Big|_{t=x/V_p} = \sin\left[\left(\frac{W(k_0)}{V_p} - k_0\right)x + \pi/4\right]$$

Therefore, the wavelength of this function is

$$\lambda = \frac{2\pi}{W(k_0)/V_p - k_0} \tag{16a}$$

Since the frequency $W(k)$ is related to the phase velocity $v(k)$

$$v(k) = W(k)/k$$

Equation (16a) can be written

$$\lambda = \frac{2\pi}{k_0\left(\frac{v(k_0)}{V_p} - 1\right)} \tag{16b}$$

where $v(k_0)$ is the phase velocity of a wave whose group velocity is V_p.

To use the asymptotic evaluation (Eq. (15)) or either of the above expressions for λ, it is necessary to determine k_0, the wave number corresponding to a wave which has group velocity V_p. Thus,

$$V_p = W'(k_0) = 2k_0^3\sqrt{\frac{D}{\rho h(k_0^4+Eh/R^2D)}} \tag{17}$$

Equation (17) can be solved for $k_0(V_p)$. There is only one positive real value of k_0 for any real value of V_p which is

$$k_0 = \left\{\left(\frac{m^3V_p^6 + 216D^2\gamma^4mV_p^2}{1728D^3} + \frac{\gamma^2mV_p^2\sqrt{(m^2V_p^4+108D^2\gamma^4)}}{48\sqrt{(3)}D^2}\right)^{1/3}\right.$$

$$\left. + \frac{m^2V_p^4}{144D^2\left(\frac{m^3V_p^6 + 216D^2\gamma^4mV_p^2}{1728D^3} + \frac{\gamma^2mV_p^2\sqrt{(m^2V_p^4+108D^2\gamma^4)}}{48\sqrt{(3)}D^2}\right)} + \frac{mV_p^2}{12D}\right\}^{1/2} \tag{18}$$

where $\gamma^4 = Eh/R^2D$ and $m = \rho h$. (Isn't MACSYMA nice?)

SOLUTION BY FOURIER TRANSFORMS

In 1963 Sing-chih Tang (ref 8) solved the related problem of a moving pressure in a semi-infinite tube through the use of Fourier transforms. Until reflected waves from the muzzle begin to influence results, Tang's solution can be used. The system solved by Tang was the same as Eqs. (1a), (1b), the first and third of Eq. (1c), and the condition that the radial displacement $w(x,t)$ remain bounded as $x \to \infty$. Tang found that the radial displacement of the tube wall in response to a moving pressure entering the tube at some constant velocity, V_p, is

$$W(X,T) = W_1(X-VT) + W_2(X+VT) - \frac{P}{q^2} e^{-\lambda_1 X} \cos(\lambda_1 X)$$

$$\frac{-2\delta^2 P}{\pi} \int_0^\infty \left(\frac{1}{\Omega^2} - \frac{1}{\Omega^2 - V^2K^2}\right) \frac{\cos(\Omega T)\sin(KX)}{K}\, dK \qquad (19)$$

where

$$W_1(\xi) = \frac{P}{q^2} \begin{cases} 1 - \tfrac{1}{2}e^{m\xi}\left[\cos(n\xi) + \frac{n^2-m^2}{2mn}\sin(n\xi)\right] , & \xi \leqslant 0 \\ \tfrac{1}{2}e^{-m\xi}\left[\cos(n\xi) - \frac{n^2-m^2}{2mn}\sin(n\xi)\right] , & \xi \geqslant 0 \end{cases}$$

$$W_2(\xi) = \frac{P}{2q^2} e^{-m\xi}\left(\cos(n\xi) - \frac{n^2-m^2}{2mn}\sin(n\xi)\right)$$

$K = n + im$ is the root of the dispersion relation

$$K^4 - V^2K^2 + \delta^2 q^2 = 0$$

Tang's variables (dimensionless) are related to those defined in the previous sections as follows:

---

[8]Sing-chih Tang, "Dynamic Response of a Thin-Walled Cylindrical Tube Under Internal Moving Pressure," Doctoral Dissertation, University of Michigan, Ann Arbor, MI, 1963, pp. 64-66.

$$X = \sqrt{12}\, x/h \qquad v_d^2 = E/(1-\nu^2)\rho \qquad V = V_p/v_d$$

$$W = w/h \qquad q^2 = \frac{E}{12\kappa G}\left(\frac{h}{R}\right)^2 \qquad \Omega^2 = K^4+\delta^2 q^2$$

$$T = \sqrt{12}\, v_d t/h \qquad \delta^2 = \frac{(1-\nu^2)\kappa G}{E} \qquad \lambda^2 = \frac{\delta q}{2}$$

$$P = p/12\kappa G \qquad G = \frac{E}{2(1+\nu)} \tag{20}$$

where κ is the shear correction factor. From Reference 1, $\kappa^2 = 0.86$ when $\nu = 0.3$; v_d is the speed of dilatational waves in a plate; K is the dimensionless wave number (wave number * $h/\sqrt{12}$); p is the magnitude of the moving pressure, and V is the dimensionless load velocity. One immediate advantage of Tang's solution is the explicit appearance of the steady-state solution shown in Figure 1. Except for W_1 and the term containing the integral, the remaining terms in Eq. (19) are needed to satisfy the boundary condition $w(0,t) = 0$ and have negligible effects elsewhere provided the velocity of the moving pressure is not too close to the critical value. The term containing the integral is the transition transient. Thus, the two main terms in the solution in regions of the tube not too close to the entrance are once again the steady-state solution corresponding to a moving pressure in an infinite tube and the transition term.

Tang was not able to evaluate the integral in closed form and proceeded with a numerical evaluation. This evaluation gave no further qualitative information, however. In the following, the method of stationary phase is applied to obtain an asymptotic evaluation of this integral. Substituting

$$\cos(\Omega T)\sin(KX) = \tfrac{1}{2}\,\mathrm{Im}\left\{\exp[i(KX+\Omega(K)T)] + \exp[i(KX-\Omega(K)T)]\right\}$$

[1]T. E. Simkins, "Resonance of Flexural Waves in Gun Tubes." ARCCB-TR-87008, Benet Laboratories, Watervliet, NY, July 1987.

thus

$$\frac{-2\delta^2 P}{\pi}\int_0^\infty F(K)\cos(\Omega T)\sin(KX)dK =$$

$$\frac{-\delta^2 P}{\pi}\,\mathrm{Im}\{\int_0^\infty F(K)e^{ih_1(K)T}dK + \int_0^\infty F(K)e^{ih_2(K)T}dK\}$$

where

$$F(K) = \frac{1}{K}\left\{\frac{1}{\Omega^2} - \frac{1}{\Omega^2 - V^2K^2}\right\} \quad , \quad h_1(K) = KX/T + \Omega(K) \quad , \quad h_2(K) = KX/T - \Omega(K)$$

The first integral contributes a negligible amount since $h_1(K)$ has no stationary point in the region $K > 0$. By stationary phase

$$\frac{-\delta^2 P}{\pi}\,\mathrm{Im}\int_0^\infty F(K)e^{i(KX/T-\Omega(K))T}dK$$

$$= \frac{-\delta^2 P}{\pi}\,\mathrm{Im}\{F(K_0)\sqrt{\frac{2\pi}{T|\Omega''(K_0)|}}\exp[-i\Omega(K_0)T + iK_0X - \frac{i\pi}{4}\,\mathrm{sgn}\,\Omega''(K_0)]\}$$

$$= F(K_0)\delta^2 P\sqrt{\frac{2}{\pi T|\Omega''(K_0)|}}\sin(\Omega(K_0)T - K_0X + \frac{\pi}{4}\,\mathrm{sgn}\,\Omega''(K_0))$$

From the expression for Ω^2 (Eq. (20))

$$\Omega''(K_0) = 2K_0^2(3q^2\delta^2+K_0^4)/(q^2\delta^2+K_0^4)^{3/2} = 2K_0^2(\Omega^2(K_0)+2q^2\delta^2)/\Omega(K_0)^3 > 0$$

Thus the asymptotic evaluation of the integral in Eq. (19) can finally be written:

$$\frac{-2\delta^2 P}{\pi}\int_0^\infty F(K)\cos(\Omega T)\sin(KX)dK = F(K_0)\delta^2 P\sqrt{\frac{2}{\pi T\Omega''(K_0)}}\sin(\Omega(K_0)T - K_0X + \pi/4) \quad (21)$$

This result was also achieved independently by Flaherty and also by Putnick (refs 9,6).

---

[6]L. Putnick, Private Communication, U.S. Army ARDEC, Benet Laboratories, Watervliet, NY, October 1988.
[9]J. Flaherty, Private Communication, U.S. Army ARDEC, Benet Laboratories, Watervliet, NY, October 1988.

The dimensionless wavelength in this term, as seen by an observer travelling with the dimensionless load velocity V, is obtained by substituting T = X/V. The result is the dimensionless counterpart of Eq. (16a):

$$\text{dimensionless wavelength} = \frac{2\pi}{\left(\frac{\Omega(K_0)}{V} - K_0\right)}$$

VERIFICATION BY FINITE ELEMENTS

The foregoing analysis was based on the method of stationary phase which gives a good approximation for sufficiently large values of t (or x). While there are means for evaluating the error (ref 7), the arithmetic is not worth the effort in this case and it is much more expedient to compare results with those from finite element simulations. These simulations were programmed by M. Leach (ref 10) of this laboratory. Since the thin shell Eq. (1a) is identical with the equation for a Bernoulli-Euler beam on an elastic foundation, the ABAQUS beam element was used in the simulations. (Even though a beam element is used, a cylindrical shell is the conceptual model.) Again, the ABAQUS model reflected the XM25 properties as reported in Reference 1 and was composed of 500 elements. The overall length of Leach's model was 5 meters (196.85 inches)--increased from the 60 inches used previously to allow for a greater number of crests to be displayed in a plot of maximum strain along the tube. The model again assumed uniform wall thickness and that the pressure moved with constant velocity. Figure 8 shows the wavelike character of the maximum radial displacement distribution along the tube computed by ABAQUS for a pressure velocity

[1] T. E. Simkins, "Resonance of Flexural Waves in Gun Tubes," ARCCB-TR-87008, Benet Laboratories, Watervliet, NY, July 1987.
[7] G. B. Whitham, Linear and Nonlinear Waves, John Wiley and Sons, New York, 1974, p. 10.
[10] M. Leach, Private Communication, U.S. Army ARDEC, Benet Laboratories, Watervliet, NY, October 1988.

equal to 85.9 percent of the critical value. Note also that the amplitude of this wave attenuates with distance along the tube. No damping was included in the ABAQUS model and this attentuation is due solely to the dispersion of the transition transient.

Early comparisons of the wavelength visible in the ABAQUS maximum strain plots, such as Figure 8, with that computed from the asymptotic evaluation (Eq. (16)) differed by as much as 40 percent, depending on the number of time increments used in integrating the ABAQUS equations of motion. For example, direct measurement of the distance from crest to crest in Figure 8 (500 time increments) gives a value of 37.5 inches, whereas Eqs. (16a) or (16b) predict only 25.96 inches. Doubling the number of time steps, however, gives a value of 26.4 inches, differing from the theoretical value by only 1.7 percent.

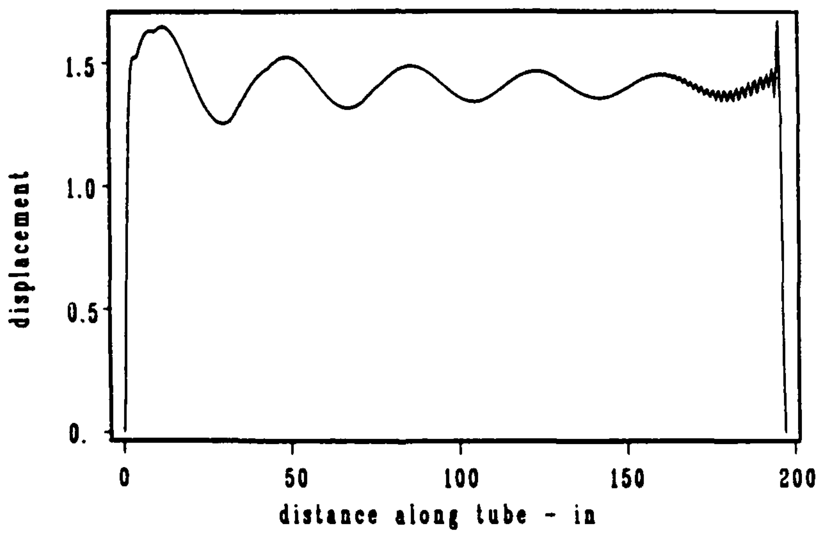

Figure 8. Maximum circumferential strain prior to shot ejection. ABAQUS (Leach).

A further example showing that the discrepancy tends to vanish as the number of time steps is increased is given in Table I. This table shows how the ABAQUS-computed wavelength converges toward a value of 10.97 inches as predicted by Eqs. (16a) and (16b) when the pressure velocity is 72 percent of the critical value.

TABLE I. EFFECT OF NUMBER OF TIME STEPS ON λ

Number of Time Steps	Wavelength* (Inches)
500	13.11
1000	11.51
2000	11.11
3000	11.00
4000	10.99
5000	10.98

*Axial distance between strain maxima.

Equation (16b) shows that as the load velocity (V_p) approaches the phase velocity, the wavelength becomes infinite. Recalling that V_p is also the group velocity of wave number k_0, this means that the phase and group velocities become equal. This is the resonant condition ($V_p = V_{cr}$) reported in Reference 1. Thus, the maximum deformation will grow as the pressure moves along the tube. The exact form of this growth must be determined from a solution valid under this resonant condition and is beyond the scope of this report.

Leach has also used the ABAQUS model to generate successive time histories of the deformation in the vicinity of the pressure front. A set of these is shown in Figure 9 when the pressure front is in the vicinity of mid-tube. The sinusoidal rise and fall of the maximum deformation is clearly visible. Figure 9b is simply a more dense version of 9a. By direct measurement, the amplitude of this sine wave is 6.29 percent of the Lamé displacement which is represented

[1]T. E. Simkins, "Resonance of Flexural Waves in Gun Tubes," ARCCB-TR-87008, Benet Laboratories, Watervliet, NY, July 1987.

by an ordinate of 1.0 in the figure. As a comparison, the stationary phase evaluation (Eq. (21)) gives the same value. Thus, the agreement is excellent.

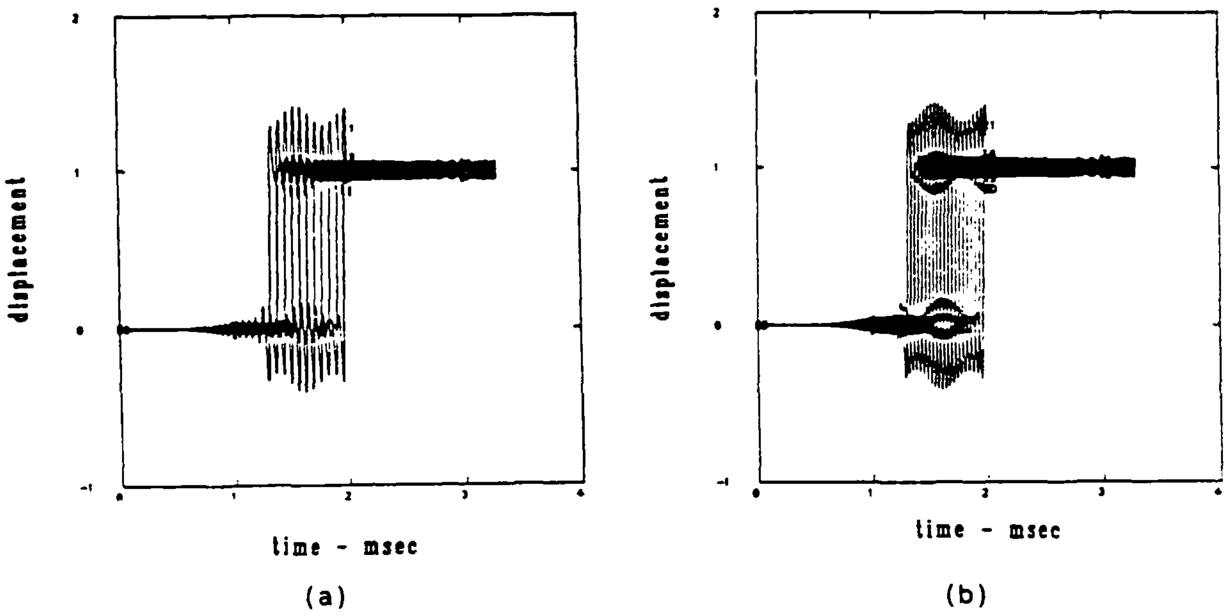

Figure 9. Successive time histories of the deformation. ABAQUS (Leach).

As previously mentioned, Leach's model employed a tube length of 5 meters in order to display several crests in his plots of maximum deformation versus distance along the tube. Because of this, the portion of the transition transient interacting with the steady deformation travelled nearly 2.5 meters by the time the pressure front reached mid-tube. Owing to the attenuation of the signal with time (or distance), the amplitude of the interaction was considerably less than it would have been had the distance travelled been less. Figure 10 shows the effect of distance travelled on the amplitude of the interference from the transition transient. For example, for a tube length of 60 inches, the amplitude was 12.0 percent of the Lamé displacement at the time the pressure reached mid-tube. Shorter tube lengths may be even more representative.

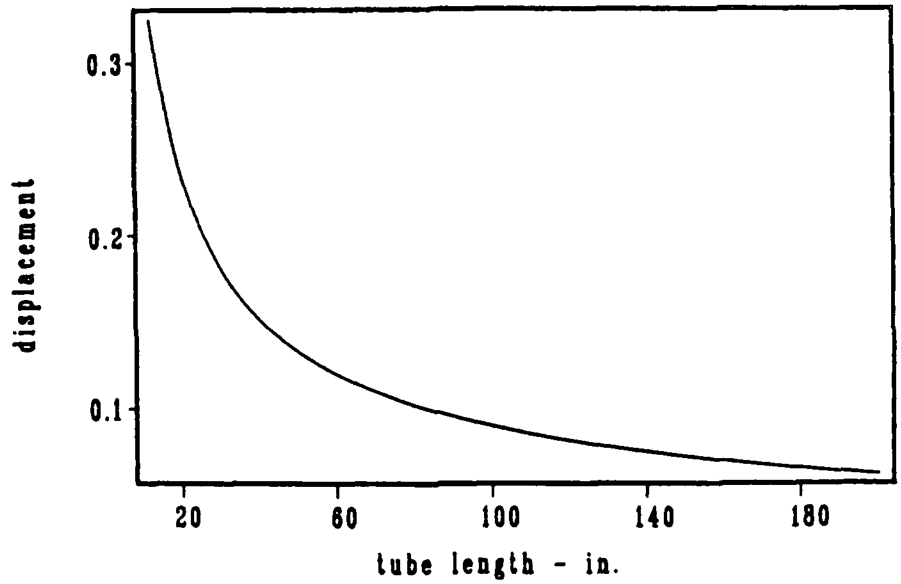

Figure 10. Effect of overall tube length on amplitude of transient deformation. Velocity of moving pressure = 0.86 V_{cr}. Pressure at mid-tube.

Finally, the growth of the sinusoidal interference as the pressure velocity approaches critical is shown in Figure 11 when the tube length was 60 inches. Larger ordinate values would result for shorter tube lengths.

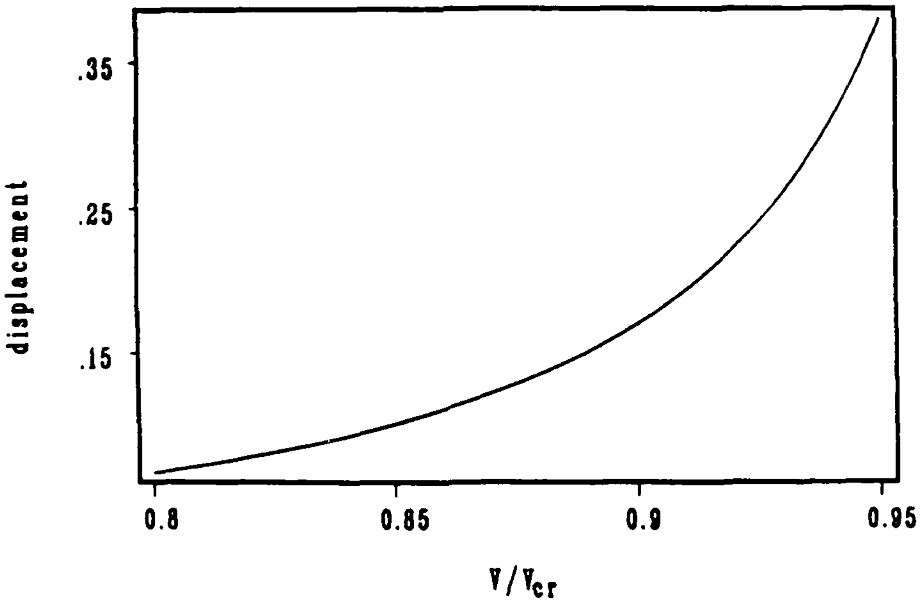

Figure 11. Effect of velocity of moving pressure on amplitude of transient deformation. Pressure at mid-tube. Tube length = 60 inches.

SUMMARY

The solution to the forced motion problem investigated in this report has been expressed as the sum of transient and steady-state motions. The transient arises as a consequence of the sudden entrance of pressure at one end of a cylinder initially undeformed and at rest--and constitutes a freely propagating deformation at all subsequent times. From the Fourier point of view, this freely propagating disturbance consists of a weighted sum of many wave trains possible in the structure (consistent with any boundary conditions). Because of dispersion, the weight of each individual wave train--and hence the deformation itself--changes with time. However--and this is important--there will be locations and times at which a few of these wave trains are nearly in phase and all of the others will be out of phase so that when the sum is performed, only these few contribute to the sum. This is shown by the method of stationary phase. Eventually these few reduce to just one so that the instantaneous deformation at any point along the cylinder essentially consists of one and only one wave train. An instant later the deformation at this point is described by a different wave train. Each wave train moves on at its corresponding group velocity. Thus, a projectile (i.e., a moving pressure front) moving at constant velocity keeps up with the specific wave train whose group velocity is equal to the projectile velocity. Generally, the phase velocity of this wave train is different from its group velocity so that its crests and valleys continually move relative to the projectile or pressure front, creating a simple harmonic deformation at this location. There is one particular wave train, however, for which the group velocity and phase velocity are equal. If the projectile is travelling at this velocity, the transient deformation will appear stationary. This is the so-called 'critical' velocity.

From the previous paragraph, one can see that the transient analysis in this report is quite general. It deals only with the free propagation of a plane disturbance through a linear material. Even within the context of the moving pressure problem, its essence has nothing to do with the axial symmetry of the problem or the constant velocity of the moving pressure, or even with the existence of the pressure itself except in providing definition to the particular disturbance being propagated. Thus, despite the specificity of the application within this report, one can safely conclude the following:

1. A sudden disturbance to a gun tube generates a transient signal which in sufficient time appears at the location of the moving projectile as a single wave, i.e., a simple harmonic deformation of the tube wall. If the location of the projectile with respect to the origin of the disturbance is denoted as X, and if the time which has passed since the disturbance occurred is T, then the group velocity of this single wave is X/T. This statement assumes only that the deformation takes place in a single mode. If the tube and pressure are perfectly axisymmetric, then the deformation is entirely axisymmetric. The wave number or wavelength and the frequency of the single wave are determined by the dispersion relation for this mode. It follows that one such wave for each mode involved in the deformation will be produced if the pressure or the geometry is asymmetric. If the projectile velocity is constant, the wavelength and frequency of the wave in each mode will be constant, but the amplitude will diminish with projectile travel. If the projectile velocity is variable, then the wavelength and frequency of the wave will be variable also and the amplitude of the wave will vary in a more general way, though in most cases of a practical nature one would still expect it to decay with projectile travel.

2. The harmonic transient deformation of the gun tube experienced in the vicinity of the projectile periodically adds to and subtracts from the steady deformation caused by the moving ballistic pressure so that the deformation is maximum at certain locations along the tube. Given sufficient time, these locations become equally spaced along the tube if the projectile velocity is constant. From the previous discussion, it is clear that the type of end conditions assumed for the tube cannot influence the spatial or temporal periodicity of this interaction, i.e., the wavelength of the interactions remains the same regardless of changes in the boundary conditions.

3. Resonance. When the pressure enters and travels through the finite tube at critical velocity, the transient deformation and the steady deformation are both unbounded terms of the solution. In addition, there are other terms normally important only near the ends of the tube which must be taken into account. However, there is no doubt that the deformation, when all terms are accounted for, must be bounded because of the finite tube length and the extent to which its amplitude increases depends on the length of the tube involved. The exact form of this growth is a subject for future work.

REFERENCES

1. T. E. Simkins, "Resonance of Flexural Waves in Gun Tubes," ARCCB-TR-87008, Benet Laboratories, Watervliet, NY, July 1987.
2. "ABAQUS Finite Element Code," Version 4.5(a), Hibbitt, Karlsson, and Sorenson, Inc., 1985.
3. A. Gabriele, Private Communication, U.S. Army ARDEC, Benet Laboratories, Watervliet, NY, October 1988.
4. Karl F. Graff, Wave Motion in Elastic Solids, Ohio State University Press, 1975, pp. 65-66.
5. G. B. Whitham, Linear and Nonlinear Waves, John Wiley and Sons, New York, 1974, p. 373.
6. L. Putnick, Private Communication, U.S. Army ARDEC, Benet Laboratories, Watervliet, NY, October 1988.
7. G. B. Whitham, Linear and Nonlinear Waves, John Wiley and Sons, New York, 1974, p. 10.
8. Sing-chih Tang, "Dynamic Response of a Thin-Walled Cylindrical Tube Under Internal Moving Pressure," Doctoral Dissertation, University of Michigan, Ann Arbor, MI, 1963, pp. 64-66.
9. J. Flaherty, Private Communication, U.S. Army ARDEC, Benet Laboratories, Watervliet, NY, October 1988.
10. M. Leach, Private Communication, U.S. Army ARDEC, Benet Laboratories, Watervliet, NY, October 1988.

TECHNICAL REPORT INTERNAL DISTRIBUTION LIST

	NO. OF COPIES
CHIEF, DEVELOPMENT ENGINEERING DIVISION	
ATTN: SMCAR-CCB-D	1
-DA	1
-DC	1
-DM	1
-DP	1
-DR	1
-DS (SYSTEMS)	1
CHIEF, ENGINEERING SUPPORT DIVISION	
ATTN: SMCAR-CCB-S	1
-SE	1
CHIEF, RESEARCH DIVISION	
ATTN: SMCAR-CCB-R	2
-RA	1
-RM	1
-RP	1
-RT	1
TECHNICAL LIBRARY	5
ATTN: SMCAR-CCB-TL	
TECHNICAL PUBLICATIONS & EDITING SECTION	3
ATTN: SMCAR-CCB-TL	
DIRECTOR, OPERATIONS DIRECTORATE	1
ATTN: SMCWV-OD	
DIRECTOR, PROCUREMENT DIRECTORATE	1
ATTN: SMCWV-PP	
DIRECTOR, PRODUCT ASSURANCE DIRECTORATE	1
ATTN: SMCWV-QA	

NOTE: PLEASE NOTIFY DIRECTOR, BENET LABORATORIES, ATTN: SMCAR-CCB-TL, OF ANY ADDRESS CHANGES.

TECHNICAL REPORT EXTERNAL DISTRIBUTION LIST

	NO. OF COPIES
ASST SEC OF THE ARMY RESEARCH AND DEVELOPMENT ATTN: DEPT FOR SCI AND TECH THE PENTAGON WASHINGTON, D.C. 20310-0103	1
ADMINISTRATOR DEFENSE TECHNICAL INFO CENTER ATTN: DTIC-FDAC CAMERON STATION ALEXANDRIA, VA 22304-6145	12
COMMANDER US ARMY ARDEC ATTN: SMCAR-AEE	1
SMCAR-AES, BLDG. 321	1
SMCAR-AET-O, BLDG. 351N	1
SMCAR-CC	1
SMCAR-CCP-A	1
SMCAR-FSA	1
SMCAR-FSM-E	1
SMCAR-FSS-D, BLDG. 94	1
SMCAR-IMI-I (STINFO) BLDG. 59 PICATINNY ARSENAL, NJ 07806-5000	2
DIRECTOR US ARMY BALLISTIC RESEARCH LABORATORY ATTN: SLCBR-DD-T, BLDG. 305 ABERDEEN PROVING GROUND, MD 21005-5066	1
DIRECTOR US ARMY MATERIEL SYSTEMS ANALYSIS ACTV ATTN: AMXSY-MP ABERDEEN PROVING GROUND, MD 21005-5071	1
COMMANDER HQ, AMCCOM ATTN: AMSMC-IMP-L ROCK ISLAND, IL 61299-6000	1

	NO. OF COPIES
COMMANDER ROCK ISLAND ARSENAL ATTN: SMCRI-ENM ROCK ISLAND, IL 61299-5000	1
DIRECTOR US ARMY INDUSTRIAL BASE ENGR ACTV ATTN: AMXIB-P ROCK ISLAND, IL 61299-7260	1
COMMANDER US ARMY TANK-AUTMV R&D COMMAND ATTN: AMSTA-DDL (TECH LIB) WARREN, MI 48397-5000	1
COMMANDER US MILITARY ACADEMY ATTN: DEPARTMENT OF MECHANICS WEST POINT, NY 10996-1792	1
US ARMY MISSILE COMMAND REDSTONE SCIENTIFIC INFO CTR ATTN: DOCUMENTS SECT, BLDG. 4484 REDSTONE ARSENAL, AL 35898-5241	2
COMMANDER US ARMY FGN SCIENCE AND TECH CTR ATTN: DRXST-SD 220 7TH STREET, N.E. CHARLOTTESVILLE, VA 22901	1
COMMANDER US ARMY LABCOM MATERIALS TECHNOLOGY LAB ATTN: SLCMT-IML (TECH LIB) WATERTOWN, MA 02172-0001	2

NOTE: PLEASE NOTIFY COMMANDER, ARMAMENT RESEARCH, DEVELOPMENT, AND ENGINEERING CENTER, US ARMY AMCCOM, ATTN: BENET LABORATORIES, SMCAR-CCB-TL, WATERVLIET, NY 12189-4050, OF ANY ADDRESS CHANGES.

TECHNICAL REPORT EXTERNAL DISTRIBUTION LIST (CONT'D)

	NO. OF COPIES
COMMANDER US ARMY LABCOM, ISA ATTN: SLCIS-IM-TL 2800 POWDER MILL ROAD ADELPHI, MD 20783-1145	1
COMMANDER US ARMY RESEARCH OFFICE ATTN: CHIEF, IPO P.O. BOX 12211 RESEARCH TRIANGLE PARK, NC 27709-2211	1
DIRECTOR US NAVAL RESEARCH LAB ATTN: MATERIALS SCI & TECH DIVISION CODE 26-27 (DOC LIB) WASHINGTON, D.C. 20375	1 1
COMMANDER AIR FORCE ARMAMENT LABORATORY ATTN: AFATL/MN EGLIN AFB, FL 32542-5434	1
COMMANDER AIR FORCE ARMAMENT LABORATORY ATTN: AFATL/MNF EGLIN AFB, FL 32542-5434	1
METALS AND CERAMICS INFO CTR BATTELLE COLUMBUS DIVISION 505 KING AVENUE COLUMBUS, OH 43201-2693	1

NOTE: PLEASE NOTIFY COMMANDER, ARMAMENT RESEARCH, DEVELOPMENT, AND ENGINEERING CENTER, US ARMY AMCCOM, ATTN: BENET LABORATORIES, SMCAR-CCB-TL, WATERVLIET, NY 12189-4050, OF ANY ADDRESS CHANGES.